Propagation of entangled light pulses through dispersing and absorbing channels

A.V. Chizhov *, E. Schmidt, L. Knöll, and D.-G. Welsch
Friedrich-Schiller-Universität Jena, Theoretisch-Physikalisches Institut
Max-Wien-Platz 1, D-07743 Jena, Germany
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The problem of decorrelation of entangled (squeezed-vacuum-type) light pulses of arbitrary shape passing through dispersive and absorbing four-port devices of arbitrary frequency response is studied, applying recently obtained results on quantum state transformation [Phys. Rev. A **59**, 4716 (1999)]. The fidelity and indices of (quantum) correlation based on the von Neumann entropy are calculated, with special emphasis on the dependence on the mean photon number, the pulse shape, and the frequency response of the devices. In particular, it is shown that the quantum correlations can decay very rapidly due to dispersion and absorption, and the degree of degradation intensifies with increasing mean photon number.

I. INTRODUCTION

Quantum-state entanglement of spatially separated systems is one of the most exciting features of quantum mechanics [1]. In particular in the rapidly developing field of quantum information processing (quantum teleportation [2], quantum cryptography [3], and quantum computing [4]), entanglement has been a subject of intense studies. Recently, continuous-variable systems have been of increasing interest [5–16].

Since entanglement is a highly nonclassical property, it is expected to respond very sensitively to environment influences, and thus it can decrease very fast. The mechanisms of decoherence are worth to be studied in detail [17,18], because they delimit possible applications. In particular, optical pulses prepared in entangled states typically propagate through optical fibers and/or pass optical instruments, such as beam splitters, mirrors, and interferometers. All these devices are built up by (dielectric) matter that always gives rise to some dispersion and absorption. As a result, the initially prepared quantum coherence is destroyed, and the question arises of what is the characteristic scale of quantum decorrelation.

In this paper the quantum decorrelation of two initially entangled light pulses that pass through optical devices is studied. The pulses are regarded as being nonmonochromatic modes of arbitrary shape, and the devices are regarded as being dispersing and absorbing four-port devices of arbitrary frequency response. The underlying theory of quantum state transformation has been developed recently [19]. It is based on a quantization procedure for the electromagnetic field in dispersing and absorbing inhomogeneous dielectrics [20–24], which is consistent with both the dissipation-fluctuation theorem and the QED canonical (equal-time) commutation relations. One interesting aspect of the theory is that given the complex refractive-index profiles [25] of the devices, which can be determined experimentally, the parameters relevant to the quantum-state transformation can be calculated without further assumptions.

In Section II the basic relations for describing the pulse propagation and the associated quantum-state transformation are given for the case when the pulses are initially prepared in a two-mode squeezed vacuum state. The fidelity and some characteristic measures of (quantum) correlations of the pulses are calculated and discussed in Section III, and some concluding remarks are given in Section IV.

II. PULSE PROPAGATION AND QUANTUM-STATE TRANSFORMATION

Let us consider the propagation of two nonclassical light pulses through lossy four-port devices of given complex refractive-index profiles [25]. Regarding the pulses as nonmonochromatic modes, we may define annihilation operators

*Permanent address: Joint Institute for Nuclear Research, Bogoliubov Laboratory of Theoretical Physics, 141980 Dubna, Moscow Region, Russia

for the two pulses in terms of the annihilation operators associated with the monochromatic modes that form the pulses [26,27],

$$\hat{a}[\eta] = \int_0^\infty d\omega\, \eta^*(\omega)\hat{a}(\omega), \tag{1}$$

$$\hat{d}[\tilde{\eta}] = \int_0^\infty d\omega\, \tilde{\eta}^*(\omega)\hat{d}(\omega), \tag{2}$$

where $\eta(\omega)$ and $\tilde{\eta}(\omega)$ are normalized functions that describe the pulse profiles,

$$\|\eta\|^2 \equiv (\eta|\eta) \equiv \int_0^\infty d\omega\, \eta^*(\omega)\eta(\omega) = 1, \tag{3}$$

$$\|\tilde{\eta}\|^2 \equiv (\tilde{\eta}|\tilde{\eta}) \equiv \int_0^\infty d\omega\, \tilde{\eta}^*(\omega)\tilde{\eta}(\omega) = 1. \tag{4}$$

Using the continuous-mode bosonic commutation relations

$$\left[\hat{a}(\omega), \hat{a}^\dagger(\omega')\right] = \delta(\omega-\omega') = \left[\hat{d}(\omega), \hat{d}^\dagger(\omega')\right] \tag{5}$$

(the other commutators being zero), it follows that

$$\left[\hat{a}[\eta], \hat{a}^\dagger[\eta]\right] = 1 = \left[\hat{d}[\tilde{\eta}], \hat{d}^\dagger[\tilde{\eta}]\right]. \tag{6}$$

Now let us assume that the pulses are initially prepared in an entangled quantum state of the type of a two-mode squeezed vacuum state

$$|\Psi_{\rm in}\rangle = \exp\left\{q^*\hat{a}[\eta]\,\hat{d}[\tilde{\eta}] - \mathrm{H.c.}\right\}|0\rangle \equiv \hat{S}\,(q)\,|0\rangle, \tag{7}$$

with $q = |q|e^{i\varphi_q}$ being the squeezing parameter. Most studies of continuous-variable systems in quantum information processing have been based on such states [6,7,12,15]. After having passed the devices, the output state of the pulses reads as

$$|\Psi_{\rm out}\rangle = \exp\left\{q^*\hat{a}'[\eta]\,\hat{d}'[\tilde{\eta}] - \mathrm{H.c.}\right\}|0\rangle \equiv \hat{S}'(q)|0\rangle, \tag{8}$$

where

$$\hat{a}'[\eta] = \int_0^\infty d\omega\, \eta^*(\omega)\hat{a}'(\omega), \tag{9}$$

$$\hat{d}'[\tilde{\eta}] = \int_0^\infty d\omega\, \tilde{\eta}^*(\omega)\hat{d}'(\omega), \tag{10}$$

and the continuous-mode operators $\hat{a}'(\omega)$ and $\hat{d}'(\omega)$ are given by [19]

$$\hat{a}'(\omega) = \sum_{i=1}^{2}\left[T_{i1}^*(\omega)\hat{a}_i(\omega) + F_{i1}^*(\omega)\hat{g}_i(\omega)\right], \tag{11}$$

$$\hat{d}'(\omega) = \sum_{i=1}^{2}\left[\tilde{T}_{i1}^*(\omega)\hat{d}_i(\omega) + \tilde{F}_{i1}^*(\omega)\hat{h}_i(\omega)\right] \tag{12}$$

[see A, equation (A5)]. In equations (11), (12) the continuous-mode operators $\hat{a}_1(\omega) \equiv \hat{a}(\omega)$ and $\hat{d}_1(\omega) \equiv \hat{d}(\omega)$ belong to the fields entering the first input ports of the two four-port devices, whereas $\hat{a}_2(\omega)$ and $\hat{d}_2(\omega)$ belong to the fields entering the second input ports, and the bosonic operators $\hat{g}_i(\omega)$ and $\hat{h}_i(\omega)$ describe excitations of the two devices. The quantities $T_{11}(\omega) \equiv T(\omega)$, $\tilde{T}_{11}(\omega) \equiv \tilde{T}(\omega)$ and $T_{21}(\omega) \equiv R(\omega)$, $\tilde{T}_{21}(\omega) \equiv \tilde{R}(\omega)$ are respectively the transmission and reflection coefficients of the four-port devices with respect to the incoming fields at the first input ports. In what follows we assume that the second input ports of the devices are unused, that is, the fields there are in the

vacuum state, the corresponding variables together with the device variables being referred to as the environment $\mathcal{E}$. Moreover, we assume that the devices are not excited.

It is convenient to represent each of the operators $\hat{a}'[\eta]$ and $\hat{d}'[\tilde{\eta}]$ in equations (9), (10) as a sum of two other independent bosonic operators,

$$\hat{a}'[\eta] = \|T\eta\|\,\hat{a}[\eta'] + (1-\|T\eta\|^2)^{1/2}\,\hat{q}_\eta\,, \tag{13}$$

$$\hat{d}'[\tilde{\eta}] = \|\tilde{T}\tilde{\eta}\|\,\hat{d}[\tilde{\eta}'] + (1-\|\tilde{T}\tilde{\eta}\|^2)^{1/2}\,\hat{p}_{\tilde{\eta}}\,, \tag{14}$$

where $\eta'(\omega) = T(\omega)\eta(\omega)/\|T\eta\|$ and $\tilde{\eta}'(\omega) = \tilde{T}(\omega)\tilde{\eta}(\omega)/\|\tilde{T}\tilde{\eta}\|$. Note that the operators $\hat{q}_\eta$ and $\hat{p}_{\tilde{\eta}}$ belong to the environment $\mathcal{E}$,

$$(1-\|T\eta\|^2)^{1/2}\,\hat{q}_\eta = \hat{a}_2[R\eta] + \sum_{i=1}^{2}\hat{g}_i[F_{i1}\eta], \tag{15}$$

$$(1-\|\tilde{T}\tilde{\eta}\|^2)^{1/2}\,\hat{p}_{\tilde{\eta}} = \hat{d}_2\left[\tilde{R}\tilde{\eta}\right] + \sum_{i=1}^{2}\hat{h}_i\left[\tilde{F}_{i1}\tilde{\eta}\right]. \tag{16}$$

Given the output quantum state $|\Psi_{\rm out}\rangle$ of the system, the (symmetric) characteristic function

$$\Phi_{\rm out}(\alpha,\beta) = \left\langle \exp\!\left(\alpha\hat{a}^\dagger[\eta'] + \beta\hat{d}^\dagger[\tilde{\eta}'] - {\rm H.c.}\right)\right\rangle_{\rm out} \tag{17}$$

can be calculated. In B it is shown that

$$\begin{aligned}\Phi_{\rm out}\,(\alpha,\beta) = \exp\Big\{ &-\tfrac{1}{2}\Big(|\alpha|^2\left[1+(\cosh 2|q|-1)\,\|T\eta\|^2\right]\\ &+|\beta|^2\left[1+(\cosh 2|q|-1)\,\|\tilde{T}\tilde{\eta}\|^2\right]\Big)\\ &-\tfrac{1}{2}\left(\alpha\beta e^{-i\varphi_q}\sinh 2|q| + \alpha^*\beta^* e^{i\varphi_q}\sinh 2|q|\right)\|T\eta\|\|\tilde{T}\tilde{\eta}\|\Big\}.\end{aligned} \tag{18}$$

Note that $\Phi_{\rm out}(\alpha,\beta)$ is a function only of the moduli $|T(\omega)|$ and $|\tilde{T}(\omega)|$ of the transmission coefficients. For $T(\omega) = \tilde{T}(\omega) = 1$, the characteristic function of the incoming fields is recognized.

III. PULSE CORRELATIONS

A measure of the entanglement of two subsystems (A) and (B) of a composed system (AB) that is prepared in some mixed state $\hat{\varrho}$ is the quantum relative entropy E [28], the quantum analog of the classical Kullback-Leibler entropy,

$$E(\hat{\varrho}) = \min_{\hat{\sigma}\epsilon\mathcal{S}}\,{\rm Tr}\,[\hat{\varrho}\,(\ln\hat{\varrho} - \ln\hat{\sigma})], \tag{19}$$

where $\mathcal{S}$ is the set of all separable quantum states the composed system can be prepared in. For pure states, $\hat{\varrho} = |\Psi\rangle\langle\Psi|$, the entanglement measure (19) reduces to the von Neumann entropy of one subsystem

$$E(\hat{\varrho}) = S_A = {\rm Tr}^{(A)}\!\left(\hat{\varrho}^{(A)}\ln\hat{\varrho}^{(A)}\right) = S_B, \qquad \hat{\varrho}^{(A)} = {\rm Tr}^{(B)}\hat{\varrho} \tag{20}$$

[${\rm Tr}^{(A)}$ (${\rm Tr}^{(B)}$), trace with respect to the subsystem A (B)]. In the case where $|\Psi\rangle$ is the two-mode squeezed vacuum state (7) it follows that

$$E(\hat{\varrho}) = S_{\rm th}(\bar{n}_{\rm sq}) = (\bar{n}_{\rm sq}+1)\,\ln(\bar{n}_{\rm sq}+1) - \bar{n}_{\rm sq}\,\ln\bar{n}_{\rm sq}\,, \tag{21}$$

with $\bar{n}_{\rm sq} = \sinh^2|q|$ being the mean number of photons in each mode. Hence, the entanglement is given by the von Neumann entropy of a thermal state of mean photon number $\bar{n} = \bar{n}_{\rm sq}$. Note that for large mean photon numbers the entanglement increases linearly with the squeezing parameter, $E \approx \ln\bar{n}_{\rm sq} \approx 4|q|$.

Unfortunately, there has been no explicit expression for calculating the entanglement measure (19) of mixed states, which are typically observed in noisy systems. Extensive numerical procedures would be indispensable in general, which dramatically grow up with increasing dimension of the Hilbert space [29]. Therefore some other correlation measures have been introduced. Although they are not purely quantum correlation measures [as is the entanglement measure (19)], they may be very helpful to gain insight into the problem of degradation of quantum correlations.

A. Fidelity

Fidelity can be regarded as being a measure of how close to each other are two (system) states in the corresponding Hilbert space. Fidelities have been used, e.g., to describe decoherence effects in the transmission of quantum information through noisy channels [30,31] and to characterize the quality of quantum teleportation [6].

When $\hat{\varrho}_{\rm in} = |\Psi_{\rm in}\rangle\langle\Psi_{\rm in}|$ and $\hat{\varrho}_{\rm out} = |\Psi_{\rm out}\rangle\langle\Psi_{\rm out}|$ are respectively the input and the output density operators of the overall system (composed of the two pulses and the environment), then the density operators of the incoming and outgoing fields are respectively

$$\hat{\varrho}_{\rm in}^{(a,d)} = {\rm Tr}^{(\mathcal{E})}\,\hat{\varrho}_{\rm in} \tag{22}$$

and

$$\hat{\varrho}_{\rm out}^{(a,d)} = {\rm Tr}^{(\mathcal{E})}\,\hat{\varrho}_{\rm out}\,, \tag{23}$$

(${\rm Tr}^{(\mathcal{E})}$, trace with respect to the environment). Following c, the fidelity

$$F_e = {\rm Tr}\Big(\,\hat{\varrho}_{\rm in}^{(a,d)}\,\hat{\varrho}_{\rm out}^{(a,d)}\Big) \tag{24}$$

can be defined. In C it is shown that applying the quantum state transformation outlined in Section II leads to

$$F_e = \left|1 + \Big[1 - (\eta|T\eta)(\tilde{\eta}|\tilde{T}\tilde{\eta})\Big]\;\bar{n}_{\rm sq}\right|^{-2}. \tag{25}$$

Note that F_e in equation (25) refers to the transmitted light. Replacing $T(\omega)$ and $\tilde{T}(\omega)$ with $R(\omega)$ and $\tilde{R}(\omega)$ respectively, the fidelity with respect to the reflected fields is obtained. From equation (25) it is seen that the fidelity sensitively depends on the spectral overlaps of the transmitted and the incoming pulses, and these overlaps are substantially determined by the dependence on frequency of the transmission coefficients $T(\omega)$ and $\tilde{T}(\omega)$ (cf. Fig. 4). It is worth noting that the fidelity decreases rapidly with increasing initial mean photon number, that is, with increasing initial squeezing and thus increasing initial entanglement.

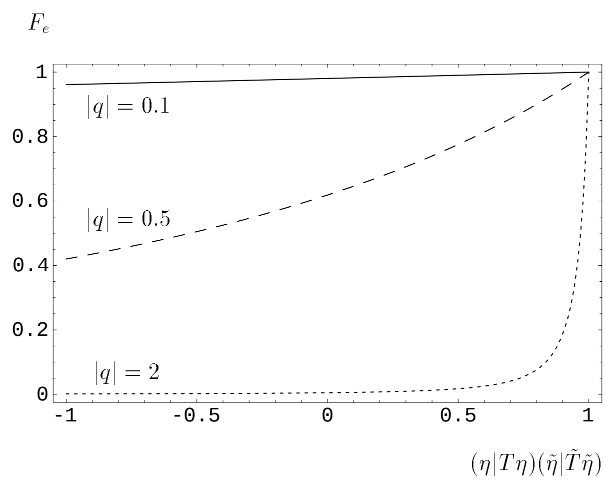

FIG. 1. The fidelity F_e, equation (25), is shown as a function of (real) $(\eta|T\eta)(\tilde{\eta}|\tilde{T}\tilde{\eta})$ for various values of the squeezing parameter $|q|$.

To illustrate the effect, F_e is shown in Fig. 1 as a function of the product of the overlaps $(\eta|T\eta)(\tilde{\eta}|\tilde{T}\tilde{\eta})$ for different values of the squeezing parameter $|q|$. In the figure $(\eta|T\eta)(\tilde{\eta}|\tilde{T}\tilde{\eta})$ is assumed to be real. If the phases of $T(\omega)$ and $\tilde{T}(\omega)$ did not depend on ω, then real $(\eta|T\eta)(\tilde{\eta}|\tilde{T}\tilde{\eta})$ would correspond to the maximally attainable fidelity (which could be also called entanglement fidelity [32]).

In quantum teleportation a strongly entangled two-mode squeezed vacuum is desired. Since in this case the photon number must be large, the state should tend to a macroscopic (at least mesoscopic) state, and hence its nonclassical features can become extremely unstable. Clearly, in the case of discrete-variable systems the entanglement must not necessarily increase with the mean photon number. For example, in the case of a two-mode state of the type

$$|\Psi_{\rm in}\rangle = \frac{1}{\sqrt{1+|\lambda|^2}}\,(|00\rangle+\lambda|nn\rangle) = \frac{1}{\sqrt{1+|\lambda|^2}}\left[1+\lambda\,\frac{\left(\hat{a}[\eta]\,\hat{d}[\tilde{\eta}]\right)^n}{n!}\right]|0\rangle\,, \tag{26}$$

the entanglement is

$$E = \frac{S_{\rm th}(|\lambda|^2)}{1+|\lambda|^2}, \tag{27}$$

whereas the mean photon number in one mode is $\bar{n} = n|\lambda|^2/(1+|\lambda|^2)$. The entanglement attains its maximal value $E = \ln 2$ at $|\lambda| = 1$, which corresponds to a Bell-type state. Nevertheless, when $\bar{n}$ becomes large the entanglement of the transmitted field decreases exponentially with $\bar{n}$ [29]. A similar behaviour is observed for the fidelity. Using the results in Ref. [29], the fidelity (with respect to the transmitted fields) is obtained to be

$$\begin{aligned} F_e = \frac{1}{(1+|\lambda|^2)^2}\bigg[&\left|1+|\lambda|^2(\eta|T\eta)^n(\tilde{\eta}|\tilde{T}\tilde{\eta})^n\right|^2 \\ &+ |\lambda|^2\left(1-\|T\eta\|^2\right)^n\left(1-\|\tilde{T}\tilde{\eta}\|^2\right)^n\bigg]. \end{aligned} \tag{28}$$

It is seen that with increasing value of n the fidelity rapidly decreases to the minimal value, i.e., $F_e = 0.5$ for $|\lambda| = 1$.

B. Entropic correlation measures

Correlation measures can be defined employing the von Neumann entropy. A very general correlation measure is the index of correlation [33]

$$I_c = S_a + S_d - S_{ad}\,, \tag{29}$$

where

$$S_{ad} = -{\rm Tr}\left(\hat{\varrho}_{\rm out}^{(a,d)}\ln\hat{\varrho}_{\rm out}^{(a,d)}\right) \tag{30}$$

is the entropy of the two-pulse system, and

$$S_m = -{\rm Tr}\left(\hat{\rho}_{\rm out}^{(m)}\ln\hat{\rho}_{\rm out}^{(m)}\right) \tag{31}$$

($m = a, d$) are the entropies of the single-pulse systems,

$$\hat{\varrho}_{\rm out}^{(a)} = {\rm Tr}^{(d)}\,\hat{\varrho}_{\rm out}^{(a,d)}, \qquad \hat{\varrho}_{\rm out}^{(d)} = {\rm Tr}^{(a)}\,\hat{\varrho}_{\rm out}^{(a,d)}. \tag{32}$$

Note that I_c is bounded from below by $|S_a - S_d|$ and from above by $S_a + S_d$. Further, I_c is an upper bound of the entanglement, $E \le I_c$, because $\hat{\varrho}_{\rm out}^{(a)} \otimes \hat{\varrho}_{\rm out}^{(d)}$ is an element of the set of separable states $\mathcal{S}$ of the two-pulse system.

Correlation measures that can be used to formulate criteria of nonclassical correlation are [30,31]

$$I_e^{(m)} = S_m - S_{ad} \tag{33}$$

($m = a, d$). Since the entropy of a classical system must not be less than the entropy of one of its subsystems, positive values of $I_e^{(m)}$ indicate nonclassical correlation. Thus, positive values of $I_e^{(a)}$ and/or $I_e^{(b)}$ may be regarded as indicating entanglement.

As can be seen from equation (18), the characteristic function of $\hat{\varrho}_{\rm out}^{(a,d)}$ is of Gaussian type. The same is true for the characteristic function of $\hat{\varrho}_{\rm out}^{(a)}$ and $\hat{\varrho}_{\rm out}^{(d)}$, which follows from equation (18) for $\beta = 0$ and $\alpha = 0$ respectively. Hence, the entropies S_m, equation (31), and [after diagonalizing the quadratic form in the exponent in equation (18)] the entropy S_{ad}, equation (30), can be obtained analytically in the form of the entropy of thermal states:

$$S_a = S_{\rm th}(n_a), \quad n_a = \|T\eta\|^2\bar{n}_{\rm sq}\,, \tag{34}$$

$$S_d = S_{\rm th}(n_d), \quad n_d = \|\tilde{T}\tilde{\eta}\|^2\bar{n}_{\rm sq}\,, \tag{35}$$

$$S_{ad} = S_{\rm th}(n_{ad}), \quad n_{ad} = (1-\|T\eta\|^2\|\tilde{T}\tilde{\eta}\|^2)\bar{n}_{\rm sq}\,. \tag{36}$$

Equations (34) – (36) again reveal the typical dependence on the spectral overlaps of the transmitted and the incoming pulses, the overlaps being substantially determined by the frequency response of the devices. Note that only the absolute values of $T(\omega)$ and $\tilde{T}(\omega)$ appear. Obviously, the entropies of the reflected light are obtained by replacing $T(\omega)$ and $\tilde{T}(\omega)$ with $R(\omega)$ and $\tilde{R}(\omega)$ respectively.

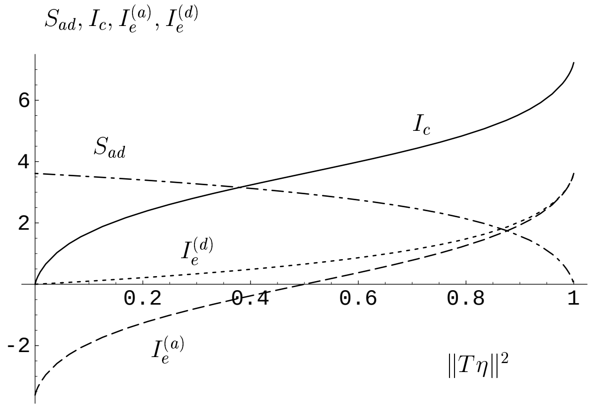

FIG. 2. The entropy S_{ad}, equation (30), and the correlation indices I_c, equation (29), and $I_e^{(a)}$ and $I_e^{(d)}$, equation (33), are shown as functions of $\|T\eta\|^2 = (T\eta|T\eta)$ for $|q| = 2$ [$\tilde{T}(\omega) = 1$].

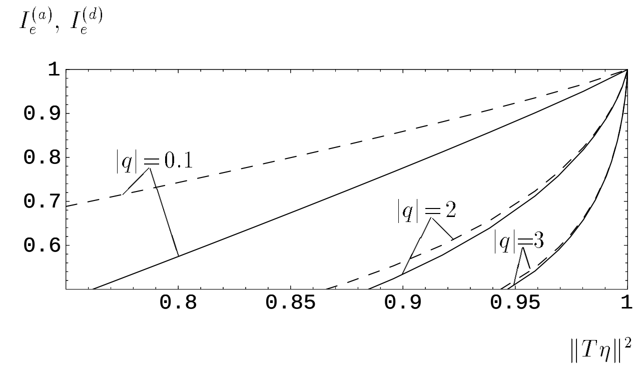

FIG. 3. The normalized correlation indices $I_e^{(a)}$ (solid line) and $I_e^{(d)}$ (dashed line), equation (33), are shown as functions of $\|T\eta\|^2 = (T\eta|T\eta)$ for various values of the squeezing parameter $|q|$ [$\tilde{T}(\omega) = 1$]. The values $|q| = 0.1$, $|q| = 2$, and $|q| = 3$, respectively, correspond to the mean photon numbers $\bar{n}_{\rm sq} = 0.01$, $\bar{n}_{\rm sq} = 13$, and $\bar{n}_{\rm sq} = 100$.

Examples of the entropy S_{ad} and the correlation indices I_c and $I_e^{(m)}$ as functions of $\|T\eta\|^2$ are shown in Fig. 2 for $\|\tilde{T}\tilde{\eta}\|^2 = 1$ (that is, only one channel is noisy). It is seen that for not too small values of $\|T\eta\|^2$ both $I_e^{(a)}$ and $I_e^{(d)}$ are positive and thus indicate entanglement. With decreasing value of $\|T\eta\|^2$ they decrease in a similar way as I_c. Whereas $I_e^{(a)}$ referring to the noise channel becomes negative and thus attains classically allowed values, $I_e^{(d)}$ referring to the unperturbed channel remains always positive but becomes small. The results are in agreement with the Peres-Horodecki separability criterion for bipartite Gaussian quantum states [16]. It tells us that in the low-temperature limit the two-pulse system under consideration remains inseparable for all values of $\|T\eta\|^2$ and $\|\tilde{T}\tilde{\eta}\|^2$ (cf. [34]). In order to get insight into the influence of the initial mean photon number $\bar{n}_{\rm sq} = \sinh^2|q|$ on the degradation of the quantum correlation, we have plotted in Fig. 3 the dependence on $\|T\eta\|^2$ of the correlation indices $I_e^{(a)}$ and $I_e^{(d)}$ for different values of $|q|$. It is clearly seen that the larger $\bar{n}_{\rm sq}$ becomes, the faster $I_e^{(a)}$ and $I_e^{(d)}$ decrease.

C. Example: absorbing dielectric plate

The values of $(\eta|T\eta)$, $(\tilde{\eta}|\tilde{T}\tilde{\eta})$, and $\|T\eta\|^2$, $\|\tilde{T}\tilde{\eta}\|^2$ depend on the chosen pulse forms and on the dependence on frequency of the transmission coefficients of the devices $T(\omega)$ and $\tilde{T}(\omega)$. When the bandwidths of the pulses are sufficiently small, so that the variation of $T(\omega)$ and $\tilde{T}(\omega)$ within the pulse bandwidths can be disregarded, then $T(\omega)$ and $\tilde{T}(\omega)$ can be taken at the mid-frequencies ω_f and $\tilde{\omega}_f$ of the pulse, i.e., $(\eta|T_{ij}\eta) \approx T_{ij}(\omega_f)$ and $(\tilde{\eta}|\tilde{T}_{ij}\tilde{\eta}) \approx \tilde{T}_{ij}(\tilde{\omega}_f)$. In this case, the results become independent of the pulse shape and solely reflect the effect of the devices.

Let us again consider the case where one pulse passes through free space and assume that the other pulse passes through a dielectric plate whose complex (Lorentz-type) permittivity is given by

$$\epsilon = 1 + \frac{\epsilon_{\rm s} - 1}{1 - (\omega/\omega_0)^2 - 2i\gamma\omega/\omega_0^2} \,. \tag{37}$$

Explicit expressions for the transmission coefficient $T(\omega)$ and the reflection coefficient $R(\omega)$ of such a device are given in Ref. [25]. In Figs. 4(a) and (b) they are shown as functions of ω for $\epsilon_{\rm s} = 1.5$, $\gamma/\omega_0 = 0.01$, and plate thickness $2c/\omega_0$. Typically, the transmission amplitude shows a dip near the medium resonance ω_0, whereas the reflection is peaked there. In Figs. 4(c) and (d), the fidelities F_e of the transmitted and reflected fields are shown as functions of ω_f. Finally, Figs. 4(e) – (h) show the dependence on ω_f of the corresponding correlation indices $I_e^{(m)}$. In particular, it is seen that for transmission F_e and $I_e^{(m)}$ are strongly reduced near the medium resonance, $\omega_f \approx \omega_0$, whereas for reflection they are enhanced in that region.

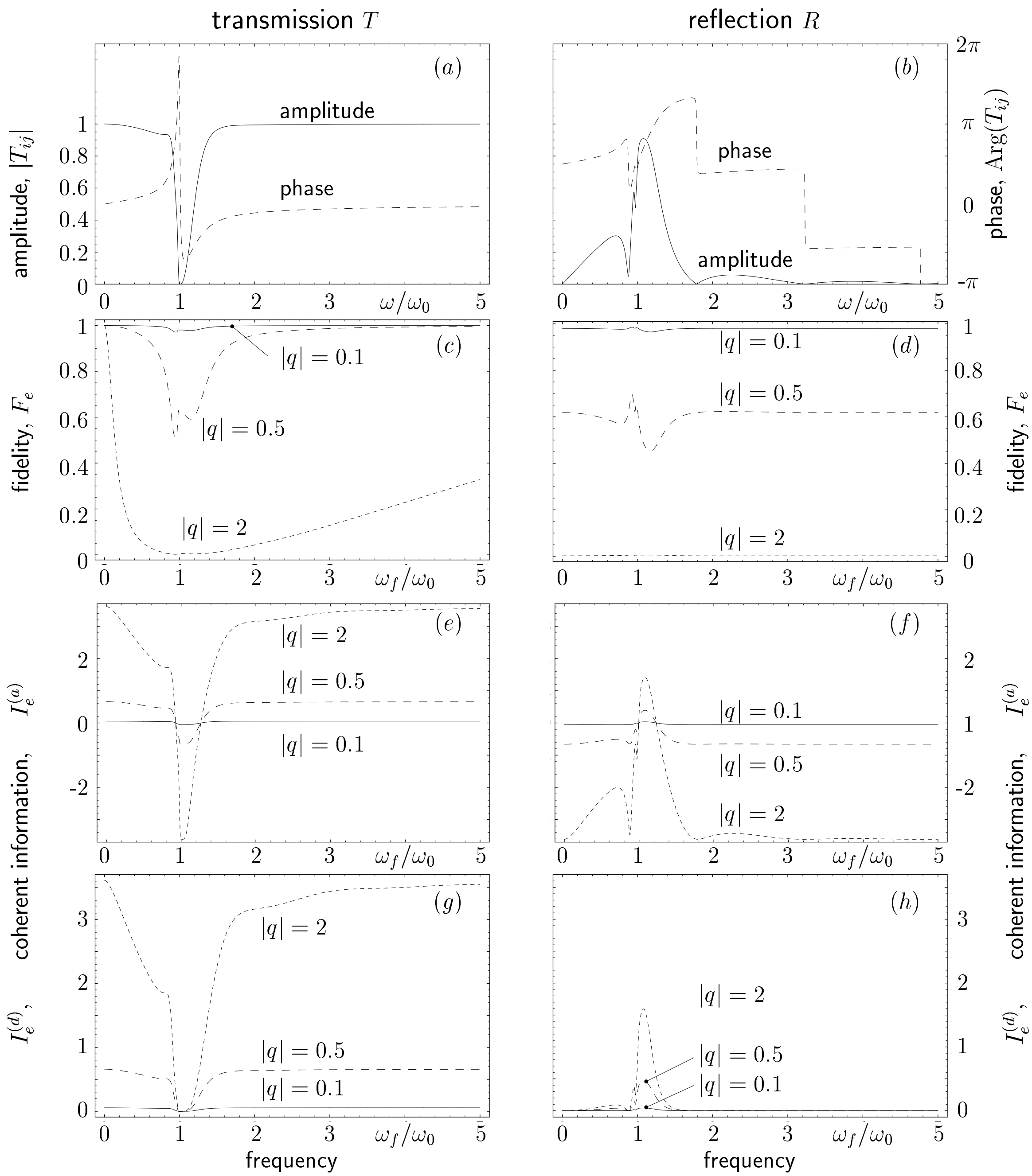

FIG. 4. The fidelity, equation (25), and the correlation indices $I_e^{(a)}$ and $I_e^{(d)}$, equation (33) are shown as functions of the mid-frequency ω_f of narrow-bandwidth pulses that are transmitted (left figures) and reflected (right figures) at a dielectric plate for various values of the squeezing parameter $|q|$ $[\tilde{T}(\omega) = 1]$. For comparison, the dependence on frequency of the amplitude and the phase of the transmission coefficient $T(\omega)$ and the reflection coefficient $R(\omega)$ of the dielectric plate [complex permittivity (37) with $\epsilon_s = 1.5$, $\gamma/\omega_0 = 0.01$; thickness $2c/\omega_0$] are shown.

IV. CONCLUSIONS

We have studied the problem of quantum correlations in a system of two entangled optical pulses of arbitrary shape, propagating through dispersing and absorbing four-port devices, which may serve as models of beam splitters, fibres, interferometers etc. Given the complex refractive-index profiles of the devices, which can be determined experimentally, the relevant parameters for the pulse propagation and the associated quantum-state transformation

can be calculated without further assumptions. The devices can be viewed as realizations of noisy channels for quantum information transmission.

In the calculations we have assumed that the pulses are initially prepared in a two-mode squeezed vacuum, which is typically considered in quantum teleportation of continuous-variable systems. We have calculated various correlation measures and studied their dependence on the pulse shape and mid-frequency, the frequency response of the devices, and the initial mean photon number. In particular, the results suggest that nonclassical correlations rapidly decrease with increasing mean photon number, which may drastically limit the effectively realizable non-classical correlation in continuous-variable systems. As a realization of a four-port device, we have considered a dielectric plate of a Lorentz-type complex permittivity in more detail.

Finally, it should be remembered that the fidelity and the correlation indices are not strict entanglement measures. Nevertheless, they may be helpful to find characteristic dependences and to estimate limits of entanglement transmission. Whereas for low-dimensional discrete-variable systems such as qubits numerical methods can be used to calculate the entanglement degradation exactly, the exploding effort prevents one from applying them to continuous-variable systems such as strongly squeezed two-mode vacuum states. It has therefore been a great challenge to find explicit entanglement measures, at least for some classes of states such as Gaussian states.

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APPENDIX A: INPUT-OUTPUT RELATIONS

The operator input-output relations at a dispersive and absorbing four-port read [25,35]

$$\hat{b}_j(\omega) = \sum_{i=1}^{2} T_{ji}\left(\omega\right) \hat{a}_i\left(\omega\right) + \sum_{i=1}^{2} A_{ji}\left(\omega\right) \hat{g}_i\left(\omega\right), \tag{A1}$$

where $\hat{a}_i(\omega)$ and $\hat{b}_j(\omega)$ are respectively the input- and output (destruction) operators of the radiation, and $\hat{g}_i(\omega)$ are the bosonic (destruction) operators of the device excitations. The 2×2 matrices $\mathbf{T}(\omega)$ (transformation matrix) and $\mathbf{A}(\omega)$ (absorption matrix), which are determined by the complex refractive-index profile of the device [25], satisfy the relation

$$\mathbf{T}(\omega)\mathbf{T}^{+}(\omega) + \mathbf{A}(\omega)\mathbf{A}^{+}(\omega) = \mathbf{I}, \tag{A2}$$

provided that the device is embedded in vacuum. Whereas the matrix $\mathbf{T}(\omega)$ describes the effects of transmission and reflection, the matrix $\mathbf{A}(\omega)$ results from the material absorption.

Let

$$\hat{\varrho}_{\rm in} = \hat{\varrho}_{\rm in}\big[\hat{\boldsymbol{\alpha}}(\omega), \hat{\boldsymbol{\alpha}}^{\dagger}(\omega)\big] \tag{A3}$$

be the overall input density operator, which is an operator functional of $\hat{\boldsymbol{\alpha}}(\omega)$ and $\hat{\boldsymbol{\alpha}}^{\dagger}(\omega)$, with $\hat{\boldsymbol{\alpha}}(\omega)$ being a "four-vector" according to

$$\hat{\boldsymbol{\alpha}}(\omega) = \left[\hat{a}_1(\omega),\, \hat{a}_2(\omega),\, \hat{g}_1(\omega),\, \hat{g}_2(\omega)\right]^{T}. \tag{A4}$$

The operator input-output relation (A1) is then equivalent to the quantum-state transformation [19]

$$\hat{\varrho}_{\rm out} = \hat{\varrho}_{\rm in}\big[\boldsymbol{\Lambda}^{+}(\omega)\hat{\boldsymbol{\alpha}}(\omega), \boldsymbol{\Lambda}^{T}(\omega)\hat{\boldsymbol{\alpha}}^{\dagger}(\omega)\big]. \tag{A5}$$

The unitary 4×4 matrix $\boldsymbol{\Lambda}(\omega)$ can be expressed in terms of the 2×2 matrices $\mathbf{T}(\omega)$ and $\mathbf{A}(\omega)$ as

$$\boldsymbol{\Lambda}(\omega) = \begin{pmatrix} \mathbf{T}(\omega) & \mathbf{A}(\omega) \\ \mathbf{F}(\omega) & \mathbf{G}(\omega) \end{pmatrix}, \tag{A6}$$

where $\mathbf{F}(\omega) = -\mathbf{S}(\omega)\mathbf{C}^{-1}(\omega)\mathbf{T}(\omega)$, $\mathbf{G}(\omega) = \mathbf{C}(\omega)\mathbf{S}^{-1}(\omega)\mathbf{A}(\omega)$, and $\mathbf{C}(\omega) = \sqrt{\mathbf{T}(\omega)\mathbf{T}^{+}(\omega)}$, $\mathbf{S}(\omega) = \sqrt{\mathbf{A}(\omega)\mathbf{A}^{+}(\omega)}$ are commuting positive Hermitian matrices with

$$\mathbf{C}^2(\omega) + \mathbf{S}^2(\omega) = \mathbf{I}. \tag{A7}$$

APPENDIX B: DERIVATION OF EQUATION (18)

Combining equations (8) and (17) yields

$$\begin{aligned}
\Phi_{\rm out}(\alpha,\beta) &= \left\langle \exp\left(\alpha\hat{a}^\dagger[\eta'] + \beta\hat{d}^\dagger[\tilde{\eta}'] - \text{H.c.}\right)\right\rangle_{\rm out} \\
&= \left\langle 0\left|\hat{S}'^\dagger\,(q)\exp\Big(\alpha\hat{a}^\dagger[\eta'] + \beta\hat{d}^\dagger[\tilde{\eta}'] - \text{H.c.}\Big)\,\hat{S}'\,(q)\right|0\right\rangle \\
&= \left\langle 0\left|\exp\Big(\alpha\hat{S}'^\dagger\,(q)\,\hat{a}^\dagger[\eta']\,\hat{S}'\,(q) + \beta\hat{S}'^\dagger\,(q)\,\hat{d}^\dagger[\tilde{\eta}']\,\hat{S}'\,(q) - \text{H.c.}\Big)\right|0\right\rangle .
\end{aligned} \tag{B1}$$

It is not difficult to prove, on applying the operator expansion theorem, that

$$\begin{aligned}
&\hat{S}'^\dagger\,(q)\,\hat{a}[\eta']\,\hat{S}'\,(q) = \\
&= \hat{a}[\eta'] - e^{i\varphi_q}\sinh|q|\|T\eta\|\,\Big(\|\tilde{T}\tilde{\eta}\|\;\hat{d}^\dagger[\tilde{\eta}'] + (1-\|\tilde{T}\tilde{\eta}\|^2)^{1/2}\,\hat{p}^\dagger_{\tilde{\eta}}\Big) \\
&+(\cosh|q|-1)\,\|T\eta\|\,\Big(\|T\eta\|\,\hat{a}[\eta'] + (1-\|T\eta\|^2)^{1/2}\,\hat{q}_\eta\Big)
\end{aligned} \tag{B2}$$

and

$$\begin{aligned}
&\hat{S}'^\dagger\,(q)\,\hat{d}[\tilde{\eta}']\,\hat{S}'\,(q) = \\
&= \hat{d}[\tilde{\eta}'] - e^{i\varphi_q}\sinh|q|\|\tilde{T}\tilde{\eta}\|\,\Big(\|T\eta\|\;\hat{a}^\dagger[\eta'] + (1-\|T\eta\|^2)^{1/2}\,\hat{q}^\dagger_{\eta}\Big) \\
&+(\cosh|q|-1)\,\|\tilde{T}\tilde{\eta}\|\,\Big(\|\tilde{T}\tilde{\eta}\|\,\hat{d}[\tilde{\eta}'] + (1-\|\tilde{T}\tilde{\eta}\|^2)^{1/2}\,\hat{p}_{\tilde{\eta}}\Big) .
\end{aligned} \tag{B3}$$

Substituting these expressions into equation (B1) and performing a normal ordering procedure, after some straightforward calculations we arrive at $\Phi_{\rm out}(\alpha,\beta)$ as given in equation (18). Note that for $T(\omega)=\tilde{T}(\omega)\equiv 1$ the characteristic function $\Phi_{\rm in}(\alpha,\beta)$ of the quantum state of the incoming fields is obtained.

APPENDIX C: DERIVATION OF EQUATION (25)

According to equations (7), (8), and (24), we may write

$$\begin{aligned}
F_e &= \text{Tr}^{(a,d)}\,\Big(\hat{\varrho}_{\rm in}^{(a,d)}\,\hat{\varrho}_{\rm out}^{(a,d)}\Big) \\
&= \text{Tr}^{(a,d)}\left\{\hat{S}(q)|0_a,0_d\rangle\langle 0_a,0_d|\hat{S}^\dagger(q)\text{Tr}^{(\mathcal{E})}\Big[\hat{S}'(q)|0\rangle\langle 0|\hat{S}'^\dagger(q)\Big]\right\} \\
&= \text{Tr}\Big\{\hat{S}'^\dagger(q)\hat{S}(q)|0_a,0_d\rangle\langle 0_a,0_d|\hat{S}^\dagger(q)\hat{S}'(q)|0\rangle\langle 0|\Big\} \\
&= \langle 0|\hat{S}'^\dagger(q)\hat{S}(q)|0_a,0_d\rangle\langle 0_a,0_d|\hat{S}^\dagger(q)\hat{S}'(q)|0\rangle .
\end{aligned} \tag{C1}$$

We introduce the series expansions

$$\hat{S}(q)|0_a,0_d\rangle = \frac{1}{\cosh|q|}\sum_{n=0}^{\infty}\tanh^n|q|\,e^{-in\varphi_q}\,\frac{(\hat{a}^\dagger[\eta]\hat{d}^\dagger[\tilde{\eta}])^n}{n!}\,|0_a,0_d\rangle, \tag{C2}$$

$$\hat{S}'(q)|0\rangle = \frac{1}{\cosh|q|}\sum_{m=0}^{\infty}\tanh^m|q|\,e^{-im\varphi_q}\,\frac{(\hat{a}'^\dagger[\eta]\hat{d}'^\dagger[\tilde{\eta}])^m}{m!}\,|0\rangle \tag{C3}$$

and find

$$\begin{aligned}
\langle 0_a,0_d|\hat{S}^\dagger\,(q)\,\hat{S}'\,(q)\,|0\rangle &= \frac{1}{\cosh^2|q|}\sum_{n=0}^{\infty}\Bigg\{\tanh^{2n}|q| \\
&\times\langle 0_a,0_d|\,\frac{(\hat{a}[\eta]\hat{d}[\tilde{\eta}])^n}{n!}\,\frac{(\|T\eta\|\|\tilde{T}\tilde{\eta}\|\,\hat{a}^\dagger[\eta']\,\hat{d}^\dagger[\tilde{\eta}'])^n}{n!}\,|0\rangle\Bigg\}.
\end{aligned} \tag{C4}$$

Combining equations (C1) and (C4) and using the the commutation relations

$$\left[\hat{a}[\eta], \hat{a}^\dagger[\eta']\right] = (\eta|T\eta)/\|T\eta\|, \tag{C5}$$

$$\left[\hat{d}[\tilde{\eta}], \hat{d}^\dagger[\tilde{\eta}']\right] = (\tilde{\eta}|\tilde{T}\tilde{\eta})/\|\tilde{T}\tilde{\eta}\|, \tag{C6}$$

we eventually arrive at

$$\begin{aligned} F_e &= \frac{1}{\cosh^4|q|}\left|\sum_{m=0}^{\infty}\left[(\eta|T\eta)(\tilde{\eta}|\tilde{T}\tilde{\eta})\tanh^2|q|\right]^m\right|^2 \\ &= \frac{1}{\cosh^4|q|}\cdot\frac{1}{\left|1-(\eta|T\eta)(\tilde{\eta}|\tilde{T}\tilde{\eta})\tanh^2|q|\right|^2} \\ &= \left|1+\left[1-(\eta|T\eta)(\tilde{\eta}|\tilde{T}\tilde{\eta})\right]\sinh^2|q|\right|^{-2}. \end{aligned} \tag{C7}$$

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